Polar Review EC
Wk 16 Tues 2017

For the following, find $\frac{dy}{dx}$ for the given value of θ.

7. $r = 2 + 3\sin\theta$, $\theta = \frac{3\pi}{2}$

8. $r = 3(1 - \cos\theta)$, $\theta = \frac{\pi}{2}$

11. Find the point of horizontal and vertical tangency for $r = 1 + \sin\theta$. Give your answers in polar form (r, θ).

Show all work. **Calculator permitted** except unless specifically stated.

Short Answer: Sketch a graph, shade the region, and find the area.

1. one petal of $r = 2\cos(3\theta)$

2. one petal of $r = 4\sin(2\theta)$

3. interior of $r = 2 + 2\cos\theta$ (no calculator)

4. interior of $r = 2 - \sin\theta$ (no calculator)

6. inner loop of $r = 1 + 2\cos\theta$

7. between the loops of $r = 1 + 2\cos\theta$

9. inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

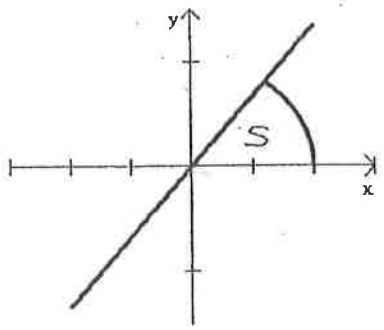
10. common interior of $r = 4\sin\theta$ and $r = 2$

13. common interior of $r = 4\sin(2\theta)$ and $r = 2$

14. inside $r = 2$ and outside $r = 2 - \sin\theta$

Free Response

16. The figure shows the graphs of the line $y = \frac{2}{3}x$ and the curve C given by $y = \sqrt{1 - \frac{x^2}{4}}$. Let S be the region in the first quadrant bounded by the two graphs and the x-axis. The line and the curve intersect at point P.



(a) Find the coordinates of P.

(b) Set up and evaluate an integral expression with respect to x that gives the area of S.

(b) Find a polar equation to represent curve C.

(d) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle θ that gives the area of S.

17. A curve is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.

(a) Find the area bounded by the curve and the y-axis.

(b) Find the angle θ that corresponds to the point on the curve with y-coordinate -1.

(c) For what values of θ, $\pi \le \theta \le \frac{3\pi}{2}$ is $\frac{dr}{d\theta}$ positive? What does this say about r?

(d) Find the value of θ on the interval $\pi \le \theta \le \frac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is this greatest distance? Justify your answer.

18. A region R in the xy-plane is bounded below by the x-axis and above by the polar curve defined by $r = \dfrac{4}{1+\sin\theta}$ for $0 \le \theta \le \pi$.

(a) Find the area of R by evaluating an integral in polar coordinates.

(b) The curve resembles an arch of the parabola $8y = 16 - x^2$. Convert the polar equation to rectangular coordinates, and prove that the curves are the same.

(c) Set up an integral in rectangular coordinates that gives the area of R.